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Determination of the Total Angular Momentum of Residual Nuclear States from Deuteron Stripping Angular Distributions

K. R. GREIDER
Yale University, New Haven, Connecticut
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A model based on diffraction techniques yields general formulas for large-angle differential cross sections in deuteron stripping (and other rearrangement) reactions in which the entrance- and exit-channel particles are strongly absorbed. It is found that for a spin-zero target, the character of the large-angle distributions depends critically on the angular-momentum transfer L (or parity of the residual state) in an unusual way. For L even, cross sections exhibit oscillations that have twice the period of the usual forward-angle stripping oscillations, while for L odd, there is almost no oscillatory structure. Furthermore, the even-L oscillations for $L=4n$ are out of phase with those for $L=4n+2$, $n=0, 1, \cdots$. A unique determination of the total spin $J=L\pm\frac{1}{2}$ of the residual nuclear state in deuteron stripping is possible when entrance- and exit-channel spin-orbit scattering, proportional to $\boldsymbol{\sigma}\cdot\mathbf{l}$, is introduced into the diffraction model. The spin-orbit amplitude is characterized by distributions of opposite parity from the spin-independent amplitude. For the case of L odd, the spin-independent amplitude is a relatively smooth function of angle, characteristic of odd-parity distributions, while the spin-dependent amplitude exhibits the even-parity $(L\pm1)$ large-angle diffraction oscillations. The analysis for $L=1$ shows that the $J=L+\frac{1}{2}=\frac{3}{2}$ state is characterized by an $L=0$ angular distribution for the spin-dependent amplitude, while the $J=\frac{1}{2}$ state shows oscillations typical of $L=2$. Consequently, a unique phase rule is obtained for identification of the total spin J of the residual state since the large-angle oscillations for $J=\frac{3}{2}$ are out of phase with those for $J=\frac{1}{2}$. A comparison of the predictions of the model with recent experiments is also presented.

RECENTLY, several deuteron stripping experiments[1] have indicated that the shape of the product angular distributions often show a quite characteristic dependence on the total angular momentum J of the final nuclear state into which the neutron is captured as well as the more familiar dependence on the orbital angular momentum L. Because of marked differences at large angles between the $J=\frac{3}{2}$ and $J=\frac{1}{2}$ distributions for $L=1$ neutron capture, Lee and Schiffer have suggested[1] that angular distributions alone may suffice to determine both the transferred orbital angular momentum L *and* the total angular momentum J of the final nuclear state without necessitating recourse to the now standard angular correlation or polarization measurements. We have investigated this J dependence within the framework of a general diffraction model and obtain a relatively simple explanation of the Lee-Schiffer result; furthermore, we find a general phase rule that should be useful in nuclear spectroscopy for a much wider class of nuclear reactions.[2,3] In order to obtain these results two separate problems must be solved: (a) Finding an adequate diffraction model for *large-angle* scattering in inelastic absorptive reactions and (b) incorporating spin-orbit scattering into the framework of such a diffraction theory. Solutions to both these problems are obtained from a two-parameter model which we believe abstracts the essential physical principles responsible for the unusual experimental observations. Unlike the many-parameter numerical analyses, simple models like the one reported here are not meant so much to provide *exact* fits to data but are used because they lead to a clearer understanding of the physics. In addition, they allow generalization of the physical principles involved in a particular reaction to a wide class of different reactions, and in this way are also extremely useful and necessary.

As in the diffraction methods used previously[4,5] to describe nuclear reactions, we assume (1) a zero range for the internal deuteron structure, (2) a surface interaction model to describe the strong nuclear absorption of scattered particles, and (3) the usual ring locus for the transfer of angular momentum L to the nucleus. The ring locus[4,5] is defined by the intersection of the target's spherical surface and a plane which passes through its center. This plane is perpendicular to the plane of scattering and contains the momentum transfer $\mathbf{q}$ (see Fig. 1). In the usual applications the ring locus leads to a two-dimensional Fraunhofer model and is reasonable when the three-dimensional aspects of the actual problem are not important. However, for large scattering angles the three-dimensional nature of the scattering from an absorbing sphere *is* important because particles emitted from the dark (nonilluminated) side of the ring must partially traverse the absorbing sphere to be detected. Consequently, the

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[1] L. L. Lee, Jr., and J. P. Schiffer, Phys. Rev. Letters **12**, 108 (1964); see also, P. T. Andrews, R. W. Clifft, L. L. Green, and J. F. Sharpey-Schafer (to be published).

[2] R. Sherr, E. Rost, and M. E. Rickey, Phys. Rev. Letters **12**, 420 (1964). Although this paper treats deuteron pickup rather than deuteron stripping, the general results of our diffraction model should still apply.

[3] R. H. Fulmer and W. W. Daehnick, Phys. Rev. Letters **12**, 455 (1964). The diffraction theory for (d,p) reactions applies also to the (d,t) case if the spin-orbit scattering of a triton is similar to that of a proton. See also A. Blair (to be published) for the (He^3,d) case.

[4] J. S. Blair, Phys. Rev. **115**, 928 (1959), and references contained therein.

[5] A. Dar, Phys. Letters **7**, 339 (1963); E. M. Henley and D. U. L. Yu, Phys. Rev. **133**, B1445 (1964), and to be published.

scattered wave in such a model does not receive equal contributions from the illuminated and dark sides; rather it originates primarily on an illuminated crescent-shaped region on the surface of the target nearest the observer. Previous considerations[6] of such a crescent region yielded smooth nonoscillatory angular distributions for large angles. However, a more exact wave treatment of the same kind of model yields a quite different result. The semiclassical conditions of Butler, Austern, and Pearson[6] are too restrictive; when these conditions are relaxed, oscillatory distributions are indeed obtained when the scattered waves originate on only one side of the absorbing target. However, the period of oscillation is not the same as that given by the usual Fraunhofer theory.[4,5]

What is the model that predicts these oscillations? First, it limits scattered wave contributions to the surface, and approximates the radial dependence by a delta function, $\delta(r-R)$. This assumption is characteristic of diffraction models and of surface reactions in general, and will be justified in a later communication. Second, the model limits the locus of momentum transfer to an angular region defined approximately by the crescent of Fig. 1. However, by itself, a crescent is not sufficient since the scattered waves receive significant contributions from the pole caps at A and C in Fig. 1, and a strict interpretation of the crescent model gives vanishingly small contributions at the poles. Consequently the illuminated angular region is a crescent modified by the pole contributions; this is approximated for angles that are not too large by an illuminated strip that begins at the pole cap A and runs along the surface through the equator at B and down to the lower pole at C. The width of the strip increases with the scattering angle. A first-order evaluation of the scattering from such a strip is obtained for $|\mathbf{k}_p|=|\mathbf{k}_d|$ from the integral over φ along the meridian line, or over the half-ring at $\theta=\pi/2$. For large scattering angles however, the integral over the effective illuminated portion of the sphere can no longer be approximated by the half-ring and a more detailed model for the illuminated surface is required. But the basic results of the half-ring integral, i.e., the L-dependent phase differences in the oscillations of angular distributions, are not changed when the scattering angle increases. It can be shown that the large angle results differ from the half-ring predictions primarily in that the argument of the oscillatory functions of Eq. (1) no longer depends on only the momentum transfer q, but is a more complicated function of the scattering angle. It is reasonable to state from these preliminary results that the quantitative predictions for large angles depend on the specific model of the illuminated region,

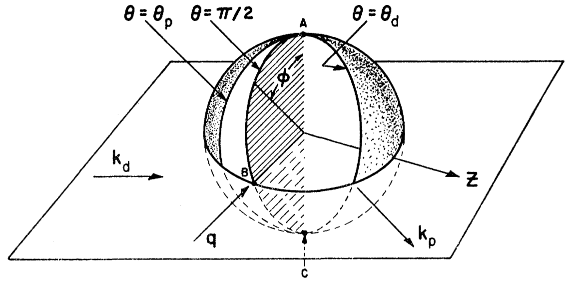

FIG. 1. The stripping reaction occurs on the spherical surface defined by $\theta_d<\theta<\theta_p$ and $0\leqslant\phi\leqslant\pi$. The angles θ_d and θ_p are obtained from the boundary conditions and define the crescent region. The integral over ϕ is evaluated at the meridian, $\theta=\pi/2$, in the shaded plane which contains $\mathbf{q}$ and which is perpendicular to the scattering plane.

while the qualitative features that permit the identification of J are essentially *independent of the model*, and are given by the solution to the half-ring locus model described above.

The φ integration over the half-ring is particularly simple when angular momentum $\hbar L$ is transferred by a spinless projectile to an initially spinless target. The matrix element for *spin-independent* large-angle scattering required in (a) consists of terms which have the form

$$f_L{}^M=P_L{}^M(\pi/2)\int_0^\pi d\varphi\, e^{-iqR\sin\varphi-iM\varphi}$$
$$=\pi P_L{}^M(\pi/2)[J_M(qR)+i\mathbf{E}_M(qR)],\quad (L-M)\text{ even},\quad (1)$$

where $\mathbf{q}=\mathbf{k}_d-\mathbf{k}_p$ is the momentum transfer and R is the nuclear radius, the radius of the ring locus. $J_M(qR)$ is the usual Bessel function of order M, and $\mathbf{E}_M$ is the Weber function defined by Watson.[7] The absolute square of Eq. (1) is plotted in Fig. 2 for $M=0$, 1, 2. Cross sections are obtained[4,5] by summing over M with the restriction that $(L-M)$ be even. It can be seen that for odd M, the cross sections are quite smooth and decrease (almost) monotonically with increasing angle. For even M, they oscillate about a smoothly decreasing background with a frequency one-half that obtained from the usual diffraction considerations. Furthermore, the $M=4n$ oscillations are out of phase with those for $M=4n+2$. This rather unusual behavior for even versus odd L (or M) provides one of the bases described below for extracting the total spin J of the final nuclear state from stripping distributions; it is, moreover, a general effect to be expected in a much broader variety of scattering phenomena.

When the entire ring locus is used as in the usual Fraunhofer model,[4,5] the integral over φ from π to 2π, which gives the complex conjugate of Eq. (1), must be

[6] S. T. Butler, N. Austern, and C. Pearson, Phys. Rev. **112**, 1227 (1958). These authors considered a more semiclassical situation in which the main contribution to the integral arises from the cylinder locus referred to in Ref. 4, and no interference can occur if one side of the nucleus is in a shadow and does not contribute to the scattered wave.

[7] G. N. Watson, *A Treatise on the Theory of Bessel Functions* (Cambridge University Press, New York, 1945), second edition, p. 308. The Weber functions are tabulated in E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, Inc., New York, 1945), p. 210. Jahnke and Emde call these functions Lommel-Weber functions and denote them by $\Omega_M(x)=-\mathbf{E}_M(x)$.

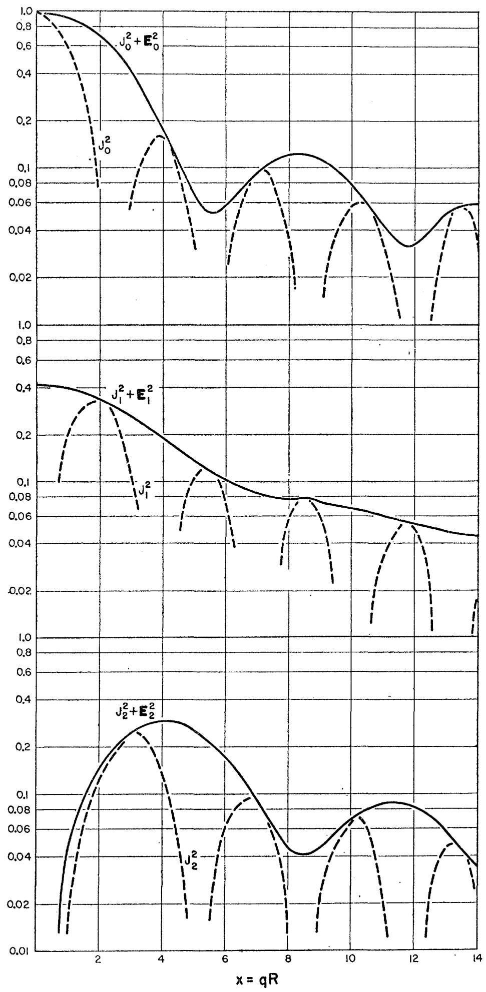

FIG. 2. The square of the Bessel function $J_M{}^2(x)$ is shown by the dashed curve. The sum of the squares of the Bessel function and the Weber function is shown by the solid curve.

added to it, and the anomalous distribution produced by the Weber function disappears. The Bessel functions that result are also shown in Fig. 2 for comparison. In general, angular distributions should be given by the dashed curve at small angles where the entire ring is required, and should gradually change to the solid curve as absorption increases at large angles. Consequently, in the transition from the dashed curve to the solid, even M distributions should show shallower minima at the odd-numbered diffraction zeros of the Bessel functions and deeper minima at the even zeros, as the scattering angle is increased.

Although Eq. (1) and Fig. 2 represent the exact solution to the half-ring model, it is worthwhile also to try to understand these distributions from a semiclassical point of view.[8] Assume that the scattering from just the three spectral points, A, B, and C of Fig. 1 represents approximately the scattering from the entire half-ring. Then for $L=0$, it is clear that a wave scattered from the "top" of the sphere at A is always *in phase* with the wave scattered from the "bottom" at C, but each travels a distance d farther than a wave scattered from the equator region (B). For $|\mathbf{k}_d|=|\mathbf{k}_p|=k$, this path difference is $d=2R\sin\theta/2$, which of course must be an integral number of wavelengths for constructive interference. Because the amplitude of waves scattered from A and C is in general different from that scattered from B, there will not be complete destructive interference, but rather finite minima will occur. The path difference d is exactly one-half that obtained using the *entire* ring locus,[4,5] where waves scattered from B and waves scattered from the *opposite* side of the sphere (or ring) combine to produce the major interference effects. As a result, the half-ring produces oscillations that have a period twice that obtained from the whole ring. These arguments reproduce the qualitative features for $M=0$ in Fig. 2.

For $M=1$, the factor $e^{-iM\varphi}$ occurring in the matrix element introduces on the ring an *intrinsic* phase difference of π between A and C so that the waves scattered from these points will interfere destructively with each other. Only the lone spectral point B contributes to the scattering, and no interference effects are expected; i.e., the $L=1$ curves should fall off smoothly as seen in Fig. 2.

For $M=2$, however, the intrinsic phase difference between A and C is 2π, and their contributions are again in phase, as they are for $M=0$. However, unlike the earlier case, the scattered wave from B now has an intrinsic phase of π relative to the $M=0$ situation, and the interference should produce long-period oscillations for $M=2$ that are *out of phase* with those for $M=0$. Such is the case, as the exact calculations of Eq. (1) show in Fig. 2. All the qualitative features of the unusual distributions of Fig. 2 are easily obtained by such semiclassical considerations. Similar arguments will be used later to help in understanding the spin dependence of the scattered wave.

Turning next to problem (b), the incorporation of spin-orbit scattering, we consider the boundary conditions introduced in Ref. 5 as producing approximate optical-model wave functions. Then spin-orbit scattering may be naturally incorporated into the theory by employing previously derived closed-form optical-model functions,[9]

$$\psi_{\text{spin orbit}}\sim[1+\alpha\boldsymbol{\sigma}\cdot\mathbf{l}]\psi_0. \qquad (2)$$

[8] The author would like to thank Dr. P. Moldauer for pointing out several of these semiclassical arguments.

[9] K. R. Greider, Nucl. Phys. 14, 498 (1960). An alternative procedure to that described in this reference is to postulate the validity of Eq. (2) on the grounds that it approximates the observed left-right asymmetry in the scattering of polarized spin-$\frac{1}{2}$ particles from spin-zero nuclei. The reason for this is that for a given spin orientation the term proportional to α has either a positive or negative value depending on whether the particle scatters on the left or right side of the nucleus. Equation (2) describes effectively a difference in the *amplitude* of the waves that scatter from the left and right sides which leads naturally to a left-right asymmetry.

ψ_0 is the optical-model wave function for scattering from a central potential. [In the present case ψ_0 is a plane wave with boundary conditions appropriate to produce a ring locus], and $\alpha=m\mu a^2/2\hbar^3$, where μa^2 is the usual strength parameter of the spin-orbit potential.

Equation (2) may be used directly in the matrix element to replace both the initial-state plane wave $e^{i\mathbf{k}_d\cdot\mathbf{r}}$, and the final-state $e^{i\mathbf{k}_p\cdot\mathbf{r}}$ of Eq. (1), and a diffraction model representation of spin-orbit scattering is obtained.

For the incoming deuteron state, there will be terms involving both the spin operator $\boldsymbol{\sigma}_p$ for the proton and $\boldsymbol{\sigma}_n$ for the neutron, while only $\boldsymbol{\sigma}_p$ appears in the final state. The complete calculation shows that the neglect of neutron spin terms does not change the qualitative features of the results, although it is required to obtain *quantitative* agreement with experiment. It is also required to obtain the *small-angle* J dependences[2] observed recently. Consequently, to minimize algebraic complexities in this report, we neglect the neutron spin-orbit scattering in what follows, and let $\boldsymbol{\sigma}_p=\boldsymbol{\sigma}$. The spin-dependent matrix element for the half-ring is

$$g_L{}^M=P_L{}^M\left(\frac{\pi}{2}\right)\int_0^\pi d\varphi\, e^{-iqR\sin\varphi-iM\varphi}\boldsymbol{\sigma}\cdot\mathbf{R}\times\boldsymbol{\Gamma}\,, \quad (3)$$

where $\boldsymbol{\Gamma}$ is the momentum sum, $\boldsymbol{\Gamma}=\mathbf{k}_d+\mathbf{k}_P$. For $|\mathbf{k}_p|=|\mathbf{k}_d|$, $\boldsymbol{\Gamma}$ lies along the z direction, and for $\theta=\pi/2$,

$$\boldsymbol{\sigma}\cdot\mathbf{R}\times\boldsymbol{\Gamma}=R\Gamma(\sigma_x\sin\varphi-\sigma_y\cos\varphi)\,. \quad (4)$$

The spin-dependent term of Eq. (3) yields Bessel and Weber functions of order $M+1$ and $M-1$ which have the opposite parity from the functions of order M obtained from the spin-independent term of Eq. (1). The algebra is carried out in the usual way with the operator expression of Eq. (3) appearing between the initial-state deuteron spin function and the final-state proton and neutron spin functions. The latter couple to L and must be described by generalized spin-angular momentum eigenfunctions. After the usual sum over final states is performed, it is found that the cross section for the $J=L-\frac{1}{2}$ state is populated more by functions of order $M+1$ than $M-1$, and the $J=L+\frac{1}{2}$ state by $M-1$ rather than $M+1$.

We consider in detail the result for the case $L=1$; explicit formulas for the cross sections at large angles are, keeping only terms through α^2,

$$\sigma_{3/2}\sim 2(J_1{}^2+\mathbf{E}_1{}^2)+(7/6)\beta^2(J_0{}^2+\mathbf{E}_0{}^2)+(5/6)\beta^2(J_2{}^2+\mathbf{E}_2{}^2)+(4/3)\sqrt{2}\beta\times[J_1(\mathbf{E}_0-\mathbf{E}_2)+\mathbf{E}_1(J_2-J_0)], \quad (4a)$$

$$\sigma_{1/2}\sim(J_1{}^2+\mathbf{E}_1{}^2)+\tfrac{2}{3}\beta^2(J_2{}^2+\mathbf{E}_2{}^2)+\tfrac{1}{3}\beta^2(J_0{}^2+\mathbf{E}_0{}^2)+\tfrac{2}{3}\sqrt{2}\beta[J_1(\mathbf{E}_0-\mathbf{E}_2)+\mathbf{E}_1(J_2-J_0)]. \quad (4b)$$

The argument of both J_M and $\mathbf{E}_M$ is $qR=2kR\sin(\theta/2)$ (for $|\mathbf{k}_p|=|\mathbf{k}_d|$), and the polarization parameter $\beta=\alpha\Gamma R=2\alpha kR\cos(\theta/2)$.

Since Eqs. (4) are valid only at large angles, the asymptotic expansion of the functions should give accurate results in this region.

$$\sigma_{3/2}\sim\left\{2\left[1+\beta^2-\frac{4}{3}\sqrt{2}\beta\right]+\frac{2}{3}\sqrt{2}\frac{\beta^2}{(\pi qR)^{1/2}}\sin\left(qR-\frac{\pi}{4}\right)\right\}\frac{1}{qR}, \quad (5a)$$

$$\sigma_{1/2}\sim\left\{\left[1+\beta^2-\frac{4}{3}\sqrt{2}\beta\right]-\frac{2}{3}\sqrt{2}\frac{\beta^2}{(\pi qR)^{1/2}}\sin\left(qR-\frac{\pi}{4}\right)\right\}\frac{1}{qR}. \quad (5b)$$

These equations are only approximate because, in the interest of clarity, we have omitted complicating terms that should be included for good quantitative results. These terms have, in fact, been calculated, and the additional algebra obtained will appear in a later communication; however, Eqs. (5) do give adequate *qualitative* predictions. In particular, these equations show that there is a clear phase difference of π between the $J=\frac{3}{2}$ and $J=\frac{1}{2}$ oscillations; this phase difference can be used in two ways to provide a *unique* prediction of J.

The first and most obvious way is by comparing the angular distribution for a reaction populating a state of unknown J with a distribution for the same energy and L value, but for which J is already known, say $J=L-\frac{1}{2}$. If the large angle oscillations are in phase, then the unknown J is the same as the known one, i.e., $J=L-\frac{1}{2}$; if they are out of phase, the unknown state must be $J=L+\frac{1}{2}$.

A second method can be used if no angular distributions for states of known J exist; this method also provides a rather critical test of some quantitative predictions of our model. Equations (5) show that the phase of the oscillation for either J is independent of the value of the spin-orbit parameter α, and depends only on qR. (This result is slightly modified in the complete calculation in which the neutron spin-orbit effects give a weak dependence of the phase on α.) The one remaining parameter R on which the phase does depend is not adjustable since it is fixed by the positions of forward-angle maxima and minima, as in the Butler theory. Consequently, the *angular* dependence of the large-angle oscillations is uniquely predicted with no free parameters. The experiments of Refs. 1 and 3 have been compared with the predictions of Eqs. (5), and there is quite good agreement in the angular positions of the large-angle maxima and minima (for $\theta<100°$); in addition, the prediction of double-period oscillations is corroborated. In all cases, the determination of the correct J value from the phase relations of Eqs. (5) is unambiguous. It should be pointed out that the

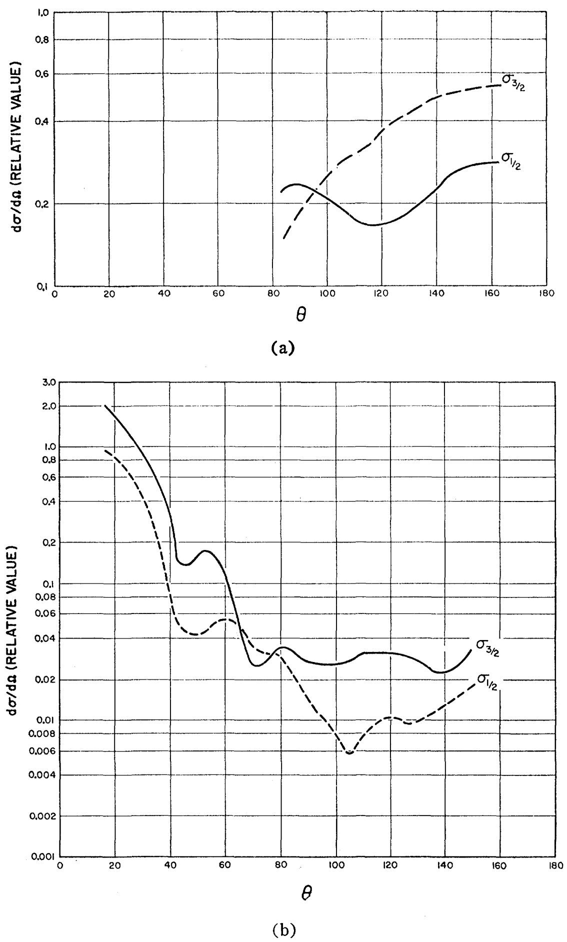

FIG. 3. (a) Calculated wide-angle stripping cross sections [see Eqs. (4)] for the $J=\frac{3}{2}$ and $J=\frac{1}{2}$ state for $L=1$ and $kR=6$. Usual diffraction effects are neglected, (i.e., $T=0$), and $\mu a^2=-9$ MeV F^2. This curve shows the typical phase differences that may be expected when there is little enhancement of the $\sigma_{1/2}$ dip. (b) Calculated stripping cross sections for the $J=\frac{3}{2}$ and $J=\frac{1}{2}$ states for $L=1$, $kR=7$, and $\mu a^2=-9$ MeV F^2. Diffraction effects are included by using Eqs. (4) and a simple classical absorption model for T: $T(\theta)=\exp(-\gamma\sin\theta/2)$, with $\gamma=3.22$. A noticeable but not the maximum enhancement of the $\sigma_{1/2}$ dip occurs at $\theta\sim104°$. These curves are shown for illustrative purposes, and no attempt has been made to "fit" any particular experiment.

criterion of Ref. 3 for determining J is based essentially on these same phase relations.

The Lee-Schiffer criterion,[1] which is related to the phase rule described above, provides yet another method of obtaining J. This method is based on their observation that in $L=1$ stripping reactions, the $\sigma_{1/2}$ cross section often exhibits a very deep minimum at a certain backward angle (either $\sim100°$ or $\sim135°$), and at a particular deuteron energy, while no such effect is seen in the $\sigma_{3/2}$ cross section. J is determined according to Ref. 1 by the presence or absence of a dip at the particular angle. Due to its sensitivity to deuteron energy and to the spin-orbit strength α, as we shall see below, this method does not appear as generally useful or reliable as the phase relationships above which apply at all energies and angles. Nevertheless, the Lee-Schiffer minimum is extraordinarily interesting in its own right, independent of the J-determination problem. The unusual and totally unexpected result is that for a wide variety of experimental situations, the minimum in $\sigma_{1/2}$ occurs at the *same angle* rather than at the same value of qR, as might be expected. A *quantitative* explanation of this unusual result is obtained from the more complicated version of our model, but a qualitative explanation can be derived from Eqs. (5).

Note that the term in square brackets in Eqs. (5) does not oscillate but has a single minimum in the range $0°\leqslant\theta\leqslant180°$. Since this factor is twice as large for $\sigma_{3/2}$ as for $\sigma_{1/2}$, it is expected that the $\sigma_{1/2}$ cross sections will show more pronounced oscillations at large angles, a prediction borne out by most experiments.[1,3] Furthermore, if the minimum in the term in brackets (at $\beta^2=8/9$) occurs near the minimum of the oscillatory term, at $\sin[qR-(\pi/4)]=+1$, a rather large enhancement of the $\sigma_{1/2}$ dip occurs. This joint requirement yields

$$q/\Gamma=\tan(\theta/2)=(4n+3/2)\pi(9/8)^{1/2}\alpha/2\,,\qquad n=1,2,3,\cdots,\qquad(6)$$

and obtains a set of discrete angles that depends on both α and kR for the possible position of a large $\sigma_{1/2}$ dip. For a given value of kR, there will be at most one angle for the enhancement, since the term in brackets has only one minimum. Equation (6) neglects the neutron spin-orbit effects which our calculations have shown are necessary to obtain *quantitative* agreement with the experimental angles of the dip. However, qualitatively, Eq. (6) like the more exact expression, predicts discrete angles for the dip in $\sigma_{1/2}$. If there is a large dip at some angle for a certain kR value, then as kR is increased, the dip begins to wash out and disappear. However, it eventually reappears again at a larger angle which corresponds to a larger kR and an increase in n by unity. The discrete nature of the dip angle has been pointed out by Lee and Schiffer,[1] and constitutes a very interesting effect which seems to be the (nuclear) spinor-wave equivalent of Brewster's angle for the scattering of (light) vector waves. At these discrete angles (functions of kR) the scattering of one spin state becomes very much smaller than the other, similar to the vanishing of one vector polarization state at Brewster's angle.

There will undoubtedly be experiments in which the scattering conditions are such that our idealized half-ring model is no longer valid. For instance when contributions from the "dark side" of the nucleus to the scattered waves are important, the ordinary diffraction oscillations $[\sim J_M{}^2(x)]$ may obscure the phase differences predicted above. Dark-side contributions may be accounted for phenomenologically by adding to Eq. (1) a term proportional to $T(\theta)[J_M(qR)-i\mathbf{E}_M(qR)]$, where $T(\theta)$ is a transmission factor that is unity for $\theta=0$ and becomes small at large angles, and $\mathbf{E}_M$ appears above with the opposite sign from that in Eq. (1). Typical calculations with and without this factor are shown in Figs. 3(a) and 3(b).

It should be noted that the spin-dependent terms for $L=1$ are not characteristic of Bessel and Weber functions of order $M=1$, but rather are described by functions of opposite parity, i.e., $M=0$ and $M=2$. It is for this reason that phase differences occur in $L=1$ reactions and characterize J. However, for $L=2$, the two spin states $J=\frac{5}{2}$ and $J=\frac{3}{2}$ are primarily characterized by distributions given by Eq. (1) with $M=1$ and $M=3$, respectively. These odd-M distributions do not oscillate, and consequently the determination of J at large angles for L even does not appear feasible.

The semiclassical explanation for the change in M by ± 1 for spin-dependent terms is relatively simple. The spin-orbit scattering amplitude is proportional to $\boldsymbol{\sigma}\cdot\mathbf{R}\times\mathbf{\Gamma}$. However, the radius vector $\mathbf{R}$ at A (Fig. 1) has the opposite sign from $\mathbf{R}$ at C. This introduces into the spin term an intrinsic phase difference of π between these two spectral points. So for odd L (say $L=1$) as we have seen, the factor $e^{-iM\varphi}$ gives a phase difference of π between A and C, but the spin-orbit effects produce another phase change of π, leaving A back in phase with C and yielding distributions like those for M even. The additional characterization of the $J=L-\frac{1}{2}$ by $L+1$ distributions and $J=L+\frac{1}{2}$ by $L-1$ is apparently a purely quantum mechanical effect that arises from the algebra of the spin functions. As yet we have not found any semiclassical picture to help in understanding it better.

We have presented a somewhat simplified version of a large-angle spin-dependent diffraction theory in the expectation that the rather simple qualitative results will provide insight into the mechanisms for the unusual effects we are trying to explain. If the general qualitative predictions of this article, and if the quantitative predictions of the more complete treatment of the model are corroborated by future experiments, deuteron stripping reactions as well as other direct reaction processes should become even more important than heretofore as a tool in nuclear spectroscopy, and further, the utilization of the unusual odd-even L behavior described here might prove fruitful in other fields.

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Interaction of High-Energy Protons and Helium Ions with Niobium*†

RALPH G. KORTELING‡ AND EARL K. HYDE
Lawrence Radiation Laboratory, University of California, Berkeley, California
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Production cross sections were measured radiochemically for isotopes of niobium, zirconium, copper, nickel, and sodium produced in niobium targets bombarded with 240, 320, 500, and 720 MeV protons and with 320, 500, 720, and 880 MeV helium ions. For the proton bombardments these cross sections were also calculated by the Monte Carlo method with an electronic computer by use of the conventional two-step model of high-energy reactions. Interpolated results of a previous calculation by Metropolis and co-workers were used to simulate the effect of the initial high-energy cascade. These results were used in turn as input data for an evaporation calculation. A comparison of the theoretical yields of final products with the observed yields indicates that the theory accounts fairly well for low-deposition-energy products (niobium and zirconium isotopes), and quite well for high-deposition-energy products (copper and nickel isotopes). The theory fails completely to account for the yields of sodium isotopes, whose production must be ascribed to fragmentation, as noted previously by others. No Monte Carlo calculations were made for helium-ion-induced reactions. However, a comparison of yields of products of helium-ion- and proton-induced reactions shows a remarkable similarity at all energies and for all products. The main difference is a greater yield by a factor of two in the case of helium-ion bombardments. The implications of this for the mechanism of fragmentation are discussed. During the course of this work a 15-min positron emitter was discovered and identified as Nb^{88}.

I. INTRODUCTION

ONE general method for the investigation of the interaction of high energy particles with complex nuclei is the radiochemical analysis of the heavy radioactive products of such interactions.[1] There exist many published studies of this type for a variety of targets bombarded with protons over the range of a few tens of MeV up to 27 GeV. The results have often been analyzed in terms of proposed mechanisms for the deposition of energy in the nucleus by the incident proton, and for the de-excitation of the excited nucleus by particle evaporation, by fission, or by fragmentation. There have

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† Based on a thesis submitted by R. G. Korteling to the Department of Chemistry, University of California, Berkeley, California.

‡ Present address: Department of Chemistry, Carnegie Institute of Technology, Pittsburgh, Pennsylvania.

[1] See review article by J. M. Miller and J. Hudis, Ann. Rev. Nucl. Sci. 9, 159 (1959).